SOLID STATE PHYSICS I

EXAM May 8, 2003

◇ Do not forget to write your full name and student number on each sheet.
◇ Please use separate sheets for each of the problems.
◇ The answers may be given in dutch

Problem 1

a) Give the ground state in spectroscopic notation of the following atoms, using Hund's rules:

   i. Pr^{3+} : [Xe] $4f^2$
   ii. Mn^{2+} : [Ar] $3d^5$
   iii. Tb^{3+} : [Xe] $4f^8$

   Note: Spectroscopic notation: ${}^nX_J$, where n is the spin degeneracy, $X = S, P, D, F, ...$ is the orbital angular momentum quantum number, and J is the spin-orbit angular momentum quantum number.

We apply a magnetic field B_A to iron (Fe, nominally $3d^8$). This field introduces a magnetization M, proportional to the field: $M = \chi B_A$. As a result of exchange interactions in the solid the magnetization field causes an effective local field $B_X = \lambda M$. The total effective field acting on the Fe atoms is now given by $B_{Loc} = B_A + \lambda M$. Assume $\lambda = 5000$. Furthermore for Pauli paramagnetism we have the following relations:

$$\frac{M}{B_A} = \frac{C}{T}.$$

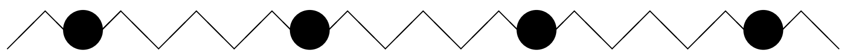
$$C = \frac{np^2\mu_b^2}{3k_b}$$

$$p = g\sqrt{S(S+1)}$$

b) Assume full quenching of the orbital angular momentum. Now calculate p for the case of iron, using $g = 2$.
c) Derive an expression for χ as a function of the temperature T.
d) Sketch χ as a function of temperature.
e) Sketch the saturation magnetization as a function of temperature.
f) Calculate the Curie temperature T_C.
g) What are magnons? Sketch the magnon dispersion relation for Fe.
h) What is, qualitatively, the influence of the magnons on the saturation magnetization.

Problem 2

Consider a linear chain consisting of identical atoms with mass M, connected by identical springs with spring constant C (see figure). Assume that the atoms can move along the chain only. Assume furthermore that each atom interacts with its nearest-neighbour atom only, and that this interaction is linear in the relative displacement along the chain.



a) What is the difference between optical and acoustical modes ? Does this chain have optical modes ?
b) Give the equation of motion of the atoms as a function of their displacement along the chain.
c) Calculate the phonon dispersion relation, and make a sketch of this.
d) Give an expression for the sound velocity along the chain.
e) Describe the physical meaning of the Debye temperature. Give an expression for the Debye temperature in the linear chain.
f) Derive an expression for the total phonon energy at low temperatures within the Debye approximation, and show that the heat capacity at low temperatures is linear in the temperature.

$$note: \qquad \int_0^\infty \frac{x}{e^x - 1}\, dx = \frac{\pi^2}{6}$$

Problem 3

In certain nonmetals like Ge, Si or Cu_2O electrons and holes may be treated in a first approximation as independent particles. Coulomb interactions may be taken into account using a simple two-particle model. An effective Hamiltonian for interacting electron - hole pairs (excitons) in a center-of-mass reference system has the form:

$$H_{eff} = -\frac{\hbar^2 \nabla^2}{2\mu} - \frac{e^2}{\epsilon r} ,$$

where $\frac{1}{\mu} = \frac{1}{m_e^*} + \frac{1}{m_h^*}$, m_e^* (m_h^*) is the electron (hole) effective mass, ϵ is the dielectric constant and $r = |\vec{r_e} - \vec{r_h}|$ The ground state wave function has the form $\Psi(\vec{r}) = \Psi_0 \exp\left(-\frac{r}{r_0}\right)$, where r_0 is the typical 'size' of the exciton.

a) Show that the expectation value for the internal kinetic energy of an exciton in the ground state is given by $\frac{\hbar^2}{2\mu r_0^2}$.
b) The Coulomb energy is given by $E_c(r_0) = -e^2/\epsilon r_0$. Derive an expression for the equilibrium size r_0 of an exciton in the ground state (*Hint:* consider the total energy as a function of the electron hole separation).

c) Cu_2O has ϵ=10, $m_e^* = 1.0\ m_e$, $m_h^* = 0.7\ m_e$. Calculate r_0 for this material (*Note:* Bohr radius $a_B = \frac{\hbar^2}{m_e e^2} = 0.53$ Å).

d) The lattice constant of Cu_2O is a=4.2 Å. Is the exciton in Cu_2O a Wannier or a Frenkel exciton ? Give arguments supporting your opinion.

e) The figure below shows the absorption spectrum of Cu_2O for energies just below the band gap $E_g = 2.172$ eV. How can you use this to test if a hydrogenic exciton model is valid for the excitons in Cu_2O?